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Collapse dynamics of trapped Bose-Einstein condensates

L. Santos[1] and G. V. Shlyapnikov[1,2,3]
[1]*Institut für Theoretische Physik, Universität Hannover, D-30167 Hannover, Germany*
[2]*FOM Institute for Atomic and Molecular Physics, Kruislaan 407, 1098 SJ Amsterdam, The Netherlands*
[3]*Russian Research Center Kurchatov Institute, Kurchatov Square, 123182 Moscow, Russia*

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We analyze the implosion and subsequent explosion of a trapped condensate after the scattering length is switched to a negative value. Our results compare very well qualitatively and fairly well quantitatively with the results of recent experiments at JILA.

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Bose-Einstein condensates (BECs) in trapped ultracold atomic gases [1–4] are strongly influenced by the atom-atom interactions. These interactions are characterized by a single parameter, the *s*-wave scattering length a. For $a>0$ the interatomic potential is repulsive and the condensate is stable. On the other hand, if $a<0$ the interaction is attractive and a uniform condensate is unstable against local collapses. The trapping potential stabilizes a condensate with a sufficiently small number of particles $N<N_c$, where N_c is a critical value below which the spacing between the trap levels exceeds the attractive mean-field interparticle interaction [5]. Trapped condensates with $a<0$ have been obtained in experiments with ^{7}Li at Rice [2], and recently at ENS [4].

Over the last decade, the creation of condensates with tunable interparticle interactions (tunable BEC) has attracted a great deal of interest. There have been several proposals on how to modify the scattering length a [6–8]. The idea of varying the magnetic field and meeting Feshbach resonances [6] has been successfully implemented for Na condensates at MIT [9]. However, the Na experiment was limited by large inelastic losses [10]. Recently, a tunable BEC of ^{85}Rb atoms has been realized at JILA, without large particle losses [3,11]. These experiments constitute an excellent tool for analyzing the influence of interatomic interactions on the BEC properties.

The JILA experiment [11] allows for the creation of large condensates with $a=a_{init}\geqslant 0$ and a subsequent sudden change of the scattering length to $a=a_{collapse}<0$. After this change of a, the condensate undergoes an implosion (collapse) followed by the ejection of relatively hot atoms (burst atoms), which form a halo surrounding a core of atoms at the trap center (remnant atoms).

The collapse of a spatially homogeneous condensate is described by the well-known nonlinear Schrödinger equation (NLSE) that in the context of BECs is called the Gross-Pitaevskii (GP) equation. This collapse has a variety of analogies, such as self-focusing of wave beams in nonlinear media, collapse of Langmuir waves, etc. (see, e.g., [12–14] and reference therein). The collapse of the solutions of NLSE has been extensively investigated and it has been found that the dimensionality of the system plays a crucial role [12,14]. In three dimensions one has a weak collapse where the singularity is reached at a finite time. Before this happens the collapsing cloud is described by a universal Zakharov solution [15]. This solution consists of quasistationary tails and a collapsing central part. The number of particles in the collapsing part continuously decreases, whereas the density increases. The dynamics of collapse in the presence of dissipation has also been analyzed (see [13,14] and reference therein). The dissipation is introduced through a nonlinear imaginary (damping) term in the NLSE, which prevents the appearance of the singularity if the nonlinearity is at least quintic, corresponding to a three-body loss process.

The collapse of a trapped condensate has been recently analyzed in several theoretical papers [16–20,29]. Kagan *et al.* [17] argued that three-body recombination should be explicitly included in the GP equation as an imaginary loss term. The recombination ‘‘burns’’ only part of the condensed atoms and prevents a further collapse of the cloud once the peak density becomes so high that the three-body loss rate is already comparable with the mean-field interaction. Then the collapse turns to expansion and the trapped condensate can undergo macroscopic oscillations accompanied by particle losses. Kagan *et al.* considered the case of a comparatively large recombination rate constant, where a single collapse does not have an internal structure. By using the formalism of Ref. [17], Saito and Ueda [19] have observed rapid intermittent implosions of the collapsing BEC cloud. This resembles the distributed collapse discussed by Vlasov *et al.* [13] in the context of collapsing cavities in plasmas. Saito and Ueda estimated the energy of the atoms released during the explosion, and predicted the formation of nonlinear patterns in the course of the collapse [29].

A different approach was suggested by Duine and Stoof [21], who proposed binary collisions as the source of burst atoms in the JILA experiments. Finally, Köhler and Burnett [22] have recently analyzed the JILA collapsing condensates with the help of a non-Markovian nonlinear Schrödinger equation, suggesting that the burst atoms can be formed due to the violation of the common *s*-wave scattering approximation.

In the present paper, we analyze the implosion and subsequent explosion of the BEC in the conditions of the recent experiments with ^{85}Rb at JILA [11]. A detailed analysis of measurable quantities is provided by numerical simulations of the GP equation, which includes three-body recombination losses as proposed in Ref. [17]. Our results agree fairly well with the data of JILA.

We consider a Bose-Einstein condensate of initially N atoms of mass m confined in a cylindrically symmetric harmonic trap. We restrict ourselves to the trap employed in ^{85}Rb experiments, i.e., a cigar-shaped trap with axial frequency $\omega_z = 2\pi\times 6.8$ Hz, and radial frequency $\omega_\rho = 2\pi \times 17.5$ Hz [3,11]. Assuming a sufficiently low temperature and omitting the presence of an initial thermal cloud, the behavior of the condensate wave function ψ is governed by the GP equation (cf. [17]):

$$i\hbar\dot{\psi} = \left(-\frac{\hbar^2\nabla^2}{2m} + V(\mathbf{r}) + g|\psi|^2 - i\frac{\hbar L_3}{12}|\psi|^4 \right)\psi, \qquad (1)$$

where $V(\mathbf{r}) = m(\omega_\rho^2\rho^2 + \omega_z^2 z^2)/2$ is the trapping potential, $g = 4\pi\hbar^2 a/m$, and ψ is normalized to N. The last term in the right-hand side of Eq. (1) describes three-body recombination losses. The quantity L_3 is the recombination rate constant for an ultracold thermal cloud, and the numerical factor 1/12 accounts for the reduction of the recombination rate by a factor of 6 in the condensate.

As in the conditions of the JILA experiment, we consider an initial scattering length $a_{init} \geqslant 0$. For this value of a we obtain the ground-state condensate wave function by evolving Eq. (1) in imaginary time. At $t=0$ the scattering length is abruptly switched to a value $a_{collapse} < 0$. The simulation of the subsequent dynamics under the conditions of JILA involves a very demanding numerical procedure, due to very different time and distance scales at $t=0$, during the collapse, and after the explosion. In our simulation we have numerically solved Eq. (1) by means of the Crank-Nicholson algorithm with variable spatial and time steps. We have taken a special care of the time and spatial numerical discretizations and checked that our results do not change significantly when a more accurate sampling is used.

An additional problem in simulating the BEC dynamics from Eq. (1) is the lack of knowledge of exact values of L_3. The experiments [23] based on the measurement of losses in thermal clouds set an upper bound for L_3, due to difficulties to distinguish between two- and three-body losses. The rate L_3 is safely determined only far from the Feshbach resonance (154.9 G for ^{85}Rb). A value of $L_3 = 4.24 \times 10^{-24}$ cm^6/s has been measured at 250 G where $a = -336a_0$ [23]. The existing predictions for L_3 are related to the case of large positive a [24,25]. For the recombination to deeply bound states, which is the case at $a<0$, the predictions contain a number of phenomenological parameters [26]. In our simulations we rely on the experimental value of L_3 at 250 G ($a=-336a_0$), and for smaller values of $|a|$ ($a<0$) we vary L_3 within the error bars of the JILA measurements [23]. This allows us to establish values of L_3 that provide a fairly good agreement between our calculations and the JILA results [11] for the dynamics of collapsing condensates.

As discussed above, the condensate collapses if N is larger than a critical value. In the absence of three-body losses, the cloud collapses continuously and the central density approaches infinity at a finite time $t_{collapse}$. After some time from the start of the collapse, the cloud becomes spherical and is described by the universal Zakharov solution [15]. The size of the central part of the cloud decreases as $(t_{collapse}-t)^{1/2}$ and the central density increases as $1/(t_{collapse}-t)$. In this stage of the collapse the presence of the trapping potential is not important (see [20]). We have tested the appearance of the universal Zakharov solution in our simulations.

The presence of three-body recombination changes the situation drastically. Once the central density becomes such that $g|\psi(0,t)|^2 \simeq (\hbar L_3/12)|\psi(0,t)|^4$, the collapse stops since the recombination losses prevent further increase of the central density [17]. We have checked that for most of realistic values of L_3 the Zakharov solution is not realized in the course of the contraction to maximum density. For comparatively small L_3 the maximum density of the cloud is rather high, and the number of particles in the central part of the cloud is small. These particles are rapidly burned by the recombination and the central density drops. However, the central region is then quickly refilled by the flux of particles from the wings of the spatial distribution. Therefore one obtains a set of intermittent collapses, i.e., the collapse becomes distributed [13,19]. As each intermittent collapse burns only a very small number of atoms the total number of particles presents a smooth time dependence.

In our calculations we have analyzed the implosion and successive explosion of a condensate for the initial number of atoms $N=6000$ and $N=15\,000$, and for $a_{collapse}$ ranging from $-25a_0$ to $-300a_0$. We have considered $a_{init}=0$ for the case of $N=6000$, and $a_{init}=7a_0$ for $N=15\,000$, in order to compare our results with those at JILA [11]. Typically, we observe that the condensate contracts mostly radially and reaches a maximum central density after a time $t_{collapse}$ that ranges from 0.5 ms to several milliseconds. Then intermittent collapses occur. Close to the maximum density in each intermittent collapse, the central region of the collapsing condensate becomes spherical. As expected, the collapse stops when $(\hbar L_3/12)|\psi(0,t)|^4 \simeq g|\psi(0,t)|^2$ [16]. At this maximum density the total number of condensed atoms (N_{tot}) decreases due to recombination losses. Due to the presence of a set of intermittent collapses, the time dependence of N_{tot} shows a stepwise decay. However, the average over short-time intervals of the order of the time interval between neighboring intermittent collapses allows us to fit N_{tot} by an exponential $\exp(-t/\tau_{decay})$. After the BEC explodes, we observe the formation of a dilute halo of burst atoms surrounding a central cloud of atoms. This reproduces qualitatively the picture observed at JILA.

We calculate various quantities: $t_{collapse}$, τ_{decay}, the number of burst (N_{burst}) and remnant ($N_{remnant}$) atoms, and the axial and radial energy of the burst atoms. We determine the number of burst atoms by integrating the condensate density over the axial coordinate and fitting the tail of the obtained radial profile by a Gaussian. The axial and radial energies per particle in the burst were calculated by averaging the axial and radial Hamiltonians, $H_z = -\hbar^2\nabla^2/2m + m\omega_z^2 z^2/2$ and $H_\rho = -\hbar^2\nabla^2/2m + m\omega_\rho^2\rho^2/2$, over the density distribution. Since the presence of the remnant cloud may introduce errors in the determination of the burst energies, we have excluded the central region from the average of H_z and H_ρ. We prevent possible errors by extracting central regions of different widths ranging from one to several harmonic-oscillator lengths. After a typical simulation time

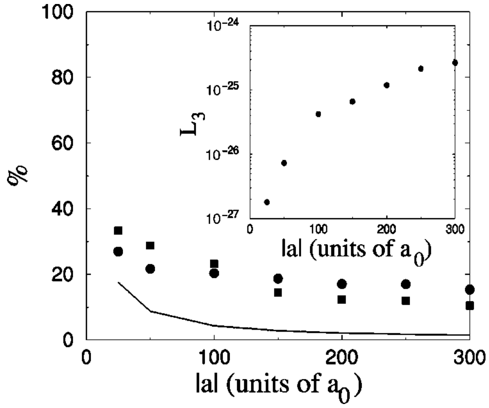

FIG. 1. The fractions N_{burst}/N (circles) and $N_{remnant}/N$ (squares) in percent vs $a_{collapse}$ for $N=6000$, $a_{init}=0$. The solid curve shows N_{cr}/N. Inset: employed L_3 values.

of 20 ms, we have checked that our results for the burst energy per particle are independent of the width of the excluded central region.

We have performed simulations for a large range of a from $-25a_0$ to $-300a_0$, varying for each a the value of L_3 within the error bars of the JILA measurement [23]. We found that within 30% of accuracy the burst energy is proportional to a^2/L_3. This scaling law was observed by Saito

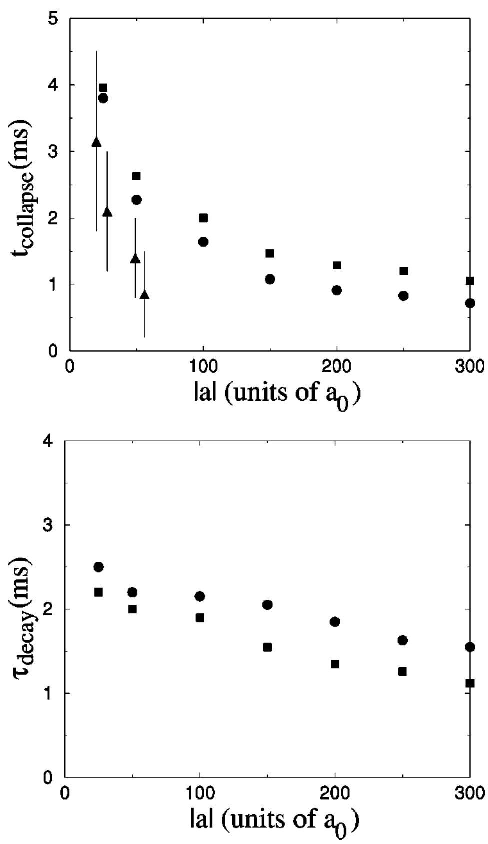

FIG. 2. The times $t_{collapse}$ (upper figure) and τ_{decay} (bottom figure) vs $a_{collapse}$ for $N=6000$, $a_{init}=0$ (circles), and $N=15\,000$, $a_{init}=7a_0$ (squares). In the upper figure the triangles show the experimental results of JILA for $N=6000$.

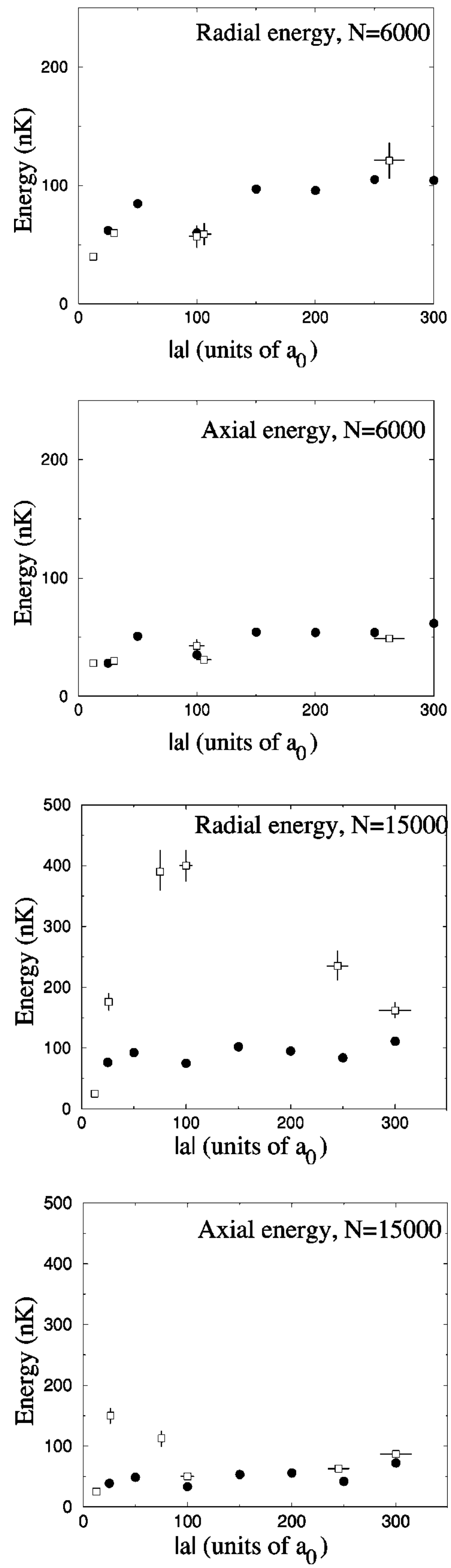

FIG. 3. Radial and axial energies vs $a_{collapse}$ for $N=6000$, $a_{init}=0$, and $N=15\,000$, $a_{init}=7a_0$. Our results (circles) are compared with the experimental data at JILA (squares with lines indicating the error bars)

and Ueda [19]. It is expected as the burst energy should be of the order of the maximum mean-field interaction $g|\psi(0,t)|^2$ at the trap center for $t \sim t_{collapse}$, and this interaction has exactly the same scaling.

As an example, we present our results for the values of L_3 that give a good fit with the JILA data on both axial and radial burst energies for $N=6000$. These values are shown in the inset of Fig. 1. In this figure we present the fractions N_{burst}/N and $N_{remnant}/N$ vs $a_{collapse}$ for $N=6000$ at a time of 20 ms. After this time N_{burst} and $N_{remnant}$ reach stationary values. In the same figure we depict the critical value $N_{cr} = k a_{ho}/|a_{collapse}|$, were $k=0.46$ is the stability coefficient for ^{85}Rb [27], and $a_{ho}=\sqrt{\hbar/m\bar{\omega}}$ with $\bar{\omega}=(\omega_\rho^2\omega_z)^{1/3}$. A similar picture has been obtained for $N=15\,000$. The burst fraction N_{burst}/N varies between 15% and 25% and only weakly depends on N and $a_{collapse}$. This is in good agreement with the results of JILA [11], where $N_{burst}/N \simeq 20\%$. One can also see that $N_{remnant} > N_{cr}$, which is expected and is in agreement with the experiments at JILA.

Figure 2 displays the dependence of $t_{collapse}$ and τ_{decay} on $a_{collapse}$ for $N=6000$ and $N=15\,000$, respectively. As observed, neither characteristic time changes significantly with changing N. The time of collapse ranges from 0.5 to 4 ms for considered values of $a_{collapse}$. The decay time τ_{decay} weakly depends on $a_{collapse}$. For $N=6000$ it ranges from 2.5 ms at $a_{collapse}=-25a_0$ to 1.6 ms at $a_{collapse}=-300a_0$. For the same values of $a_{collapse}$ at $N=15\,000$, the time τ_{decay} ranges from 2.2 to 1.1 ms. These results are in excellent agreement with Ref. [11].

Figure 3 shows the radial and axial burst energies versus $a_{collapse}$ for $N=6000$ and $N=15\,000$. Both energies smoothly depend on $a_{collapse}$ and N. The radial energy is of the order of 100 nK, and the axial one of the order of 50 nK. The values of L_3 from the inset of Fig. 1, which lead to a good agreement with the JILA results for $N=6000$, also fit fairly well with the JILA data for $N=15\,000$. The discrepancy is less than 50%, except for the radial energy at $a_{collapse} \simeq 100a_0$ where the result of the calculations is by a factor of 4 smaller than that found experimentally [28]. The discrepancy could be due to not completely well-established definition of the burst cloud.

To summarize, we have analyzed the collapse dynamics of trapped condensates after the scattering length is switched to a negative value. Our analysis, based on the GP equation with three-body losses explicitly included, explains qualitatively and to a large extent quantitatively the experiments performed at JILA.

Note added. Recently we learned that Saito and Ueda extended their analysis, also based on the GP equation with three-body losses, to the case of axially symmetric trap of JILA (second version of Ref. [29]). They calculated $t_{collapse}$ and found the number of burst and remnant atoms as functions of the initial number of atoms N at a given value of $a_{collapse}$. Their results for $t_{collapse}$ agree very well with ours.

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